FORMATION OF A SHOCK WAVE AT THE BOUNDARY OF A LOCAL SUPERSONIC ZONE

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A self-similar solution of the equation of plane potential flow has been obtained in the region of a point f where a shock wave, which limits the local supersonic zone, is formed on a sonic line. According to this solution there is a shock wave of variable intensity at the boundary of the local supersonic zone, and a characteristic is obtained at the point f on which the derivatives of the gas-dynamic parameters with respect to the coordinates are continuous. The intensity of the shock wave from the point f increases with an infinite derivative, and hence, to construct a solution, asymptotic relations on the shock wave are analyzed. The self-similarity factor varies over the range $0.3975 \leq k \leq 0.4166$, and the minimum error which is introduced by taking the quadratic terms into account corresponds to $k = 0.4017$.

1. A number of papers have been published on the problem of the wave structure of the local supersonic zone [1-8]. Theorems on the existence of a continuous solution in the transonic flow round plane bodies were proved in [1], and it was also pointed out that, according to the final theorem, when a transition occurs from continuous to discontinuous flow an ultrasonic wave occurs at the boundary and not inside the local supersonic zone. The mathematical techniques of the theory of discontinuous solutions in the transonic approximation was developed in [2]. Self-similar solutions of Tricomi's equation were investigated in velocity-hodograph variables, and an example was given of the solution with a closing local supersonic zone and a perpendicular sonic line with a direct jump in density. This solution turned out to be irregular inside the local supersonic zone since the streamlines here have a loop and hence are incorrect physically. The refinement of this solution by eliminating the irregularity by means of a sudden change in the rarefaction [3] is still under discussion, since asymptotically such a sudden change contradicts the principle of increase in entropy.

Various statements have been published which sometimes cancel one another out. Thus, the suggestion was made in [4], referring to unpublished results, that the structure of transonic flow considered below does not exist. However, such a structure – a jumpwise prolongation of the consolidation of a sonic line – was calculated in [5] by constructing an integral curve in the class of self-similar solutions of Karman's equation. This solution in turn requires additional analysis (see [6], p. 642).*

Self-similar solutions in the range considered below for the self-similarity factor were analyzed in [7, 8]. However, due to the fact that it is not possible to satisfy the shock-polar equation on the shock wave, these solutions were recognized as being physically unreal. The main difference from [7, 8] in the results derived below lies in the different representation of the boundary conditions for the self-similar solution with a shock wave, whose intensity increases from the point of formation with an infinite derivative.

2. We will give the fundamental equations and formulas of the transonic approximation of hodograph theory. Chaplygin's equation for the stream function $\psi(V, \vartheta)$ (where V and ϑ are the modulus and angle of inclination of the velocity vector) in the region of the sonic line can be converted into Tricomi's equation

$$\psi_{\eta\eta} - \eta\psi_{\tau\tau} = 0,\ \eta = (V-1)/V,\ \tau = (\varkappa + 1)^{-1/2}\vartheta. \tag{2.1}$$

Here and below for the ideal gas considered κ is Poisson's adiabatic index; the gas-dynamic quantities are reduced to dimensionless form so that we take as unity on the scale of velocity and density the critical values V_* and ρ_*, while for the pressure p we take $\rho_* V_*^2$. The independent variable τ is measured from its value on the streamline at the point f.

*The range $3/4 < k < 11/12$ is incorrectly given in [6], with reference to [5], since the result in [5] is given for $k = 2/5$.

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In the transonic approximation we have formulae for converting from the plane of the velocity hodograph to the physical x, y plane

$$dx = \psi_\tau \eta d\eta + \psi_\eta d\tau, \quad dy = d\psi. \tag{2.2}$$

In (2.2) the abscissa x is divided by $(\kappa + 1)^{1/2}$ and only those terms are retained which, in the expression for the Jacobian of the transformation, give

$$\Delta = \eta\psi_\tau^2 - \psi_\eta^2.$$

Equation (2.1) has a self-similar solution $\psi = |\tau|^{2k}F(\xi)$ ($F(\xi)$ is the general hypergeometric function (consisting of two linearly independent special solutions), $\xi = 1 - (4\eta^3)/(9\tau^2)$ is the self-similar variable, and k is the self-similarity index). In τ, ξ variables we obtain the following equations for Δ and x from (2.2):

$$\Delta = (1-\xi)^{1/3}\{[kF + (1-\xi)F']^2 - (1-\xi)F'^2\}|\tau|^{4(k-1/3)}; \tag{2.3}$$

$$x = \frac{\operatorname{sign}\tau}{6k+1}[18(1-\xi)]^{2/3}(kF - \xi F')\left(\frac{y}{F}\right)^n, \quad n = \frac{1+6k}{6k}. \tag{2.4}$$

Using Eqs. (2.4) we can map the line ξ = const into the x, y plane, where n is the self-similarity index of the solution of the equations

$$\eta\eta_x = \tau_y, \quad \eta_y = \tau_x, \tag{2.5}$$

which follow directly from (2.2). Since in the class of solutions considered the shock wave coincides in the physical plane with the line ξ = const, along its generatrix, taking the second equation of (2.2) into account, we obtain

$$\tau \sim \psi^{1/2k} \sim y^{1/2k}, \quad \eta \sim \tau^{2/3} \sim \psi^{1/3k} \sim y^{1/3k}, \tag{2.6}$$

whence it follows that at the point where the shock wave is formed the derivative of the modulus of the velocity along its generatrix is equal to infinity when $k > 1/3$, and zero when $k < 1/3$. If $k < 1/2$, the derivative of the angle of inclination τ is also equal to zero. We will denote by u and v the projections of the velocity vector on the x and y axes, respectively. On the shock wave with the equation $x = x_0 y^n$, x_0 = const, by (2.6) we obtain the relations

$$u = 1 + c_u y^{2m}, \quad p = \varkappa^{-1} + c_p y^{2m}, \quad \rho = 1 + c_\rho y^{2m}, \quad v = c_v y^{3m}, \tag{2.7}$$

where $m = n - 1$ and it is assumed that the constants c_u, c_p, c_ρ, c_v are different on both sides of the sudden jump. By calculating the mass flow G and the components of the momentum flux P_x and P_y using (2.7), we have, up to terms $\sim y^{4m}$ compared with unity,

$$G = y + \frac{c_\rho + c_u}{2m+1}y^{2m+1} + \frac{c_\rho c_u - nx_0 c_v}{4m+1}y^{4m+1},$$

$$P_x = \frac{\varkappa+1}{\varkappa}y + \frac{c_p + 2c_u + c_\rho}{2m+1}y^{2m+1} + \frac{c_u^2 + 2c_u c_\rho - nx_0 c_v}{4m+1}y^{4m+1},$$

$$P_y = \frac{x_0}{\varkappa}y^{m+1} + \frac{nx_0 c_p - c_v}{3m+1}y^{3m+1} + \frac{c_u + c_\rho}{5m+1}c_v y^{5m+1}.$$

Expanding the total enthalpy H and the entropy function $S = p\rho^{-\kappa}$ in series in powers of y we obtain

$$2H = \frac{\varkappa+1}{\varkappa-1} + 2\left(c_u + \frac{\varkappa c_p - c_\rho}{\varkappa-1}\right)y^{2m} + \left(c_u^2 + 2c_\rho\frac{c_\rho - \varkappa c_p}{\varkappa-1}\right)y^{4m},$$

$$S = \varkappa^{-1} + (c_p - c_\rho)y^{2m} - \left(\varkappa c_\rho c_p - \frac{\varkappa+1}{2}c_\rho^2\right)y^{4m}.$$

This shows that in the asymptotic expansions given above as $y \to 0$, the main terms are the terms $\sim y^{2m}$, since the terms $\sim y^{4m}$ introduce a "parasitic" effect of nonconservation of total enthalpy and give rise to vorticity.

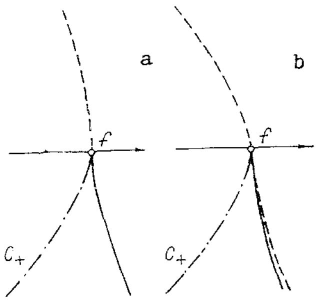

Fig. 1

Hence, from the mass, momentum and energy flow equations, we have the following three conditions on the shock wave:

$$[c_\rho + c_u] = 0, \ [c_p + 2c_u + c_\rho] = 0, \ [nx_0 c_p - c_v] = 0.$$

Here the square brackets indicate the difference between the functions enclosed within them or expressions from the left and right of the discontinuity. Hence it follows that in potential flow up to the shock wave $c_p = c_\rho = -c_u$. From the first two equations obtained on the shock wave we have that analogous equations are satisfied behind it also, i.e., vorticity of the flow does not occur in the region of the point f. Hence, in addition to the equations $[x] = 0$ and $[\psi] = 0$ or $[y] = 0$, when constructing the solution one only needs to satisfy the third of the relations written above, which denotes equality of the tangential component of the velocity on the shock wave

$$[\tau] + [\eta]x' = 0. \tag{2.8}$$

These boundary conditions on the shock wave are only obtained assuming that the self-similar solution (2.7) exists, apart from the main terms $\sim y^{2m}$, and that all the flows are equal.

If we integrate Eqs. (2.5) over the region containing the shock wave, Eq. (2.8) also follows from the second equation, while from the first we have

$$[\eta^2] + 2[\tau]x' = 0. \tag{2.9}$$

The terms in this equation are of the order of $\sim y^{4m}$, as follows directly, for example, from a comparison with the third term in the expression for G, which differs from (2.9) solely in the coefficient 2 in front of $\tau x'$. Eliminating x' from (2.9) using (2.8) we obtain an equation for the shock polar in the transonic approximation (here and below the subscripts 1 and 2 indicate parameters before and after the sudden change)

$$2(\tau_2 - \tau_1)^2 = (\eta_1 + \eta_2)(\eta_1 - \eta_2)^2. \tag{2.10}$$

It was assumed in [4, 7, 8] that when constructing the solution it is necessary to satisfy (2.10) exactly, and since this could not be done, it was asserted that a shock wave cannot occur on the sonic lines, if a weak discontinuity (a characteristic on which the derivatives of u, v, p, and ρ with respect to the coordinates are discontinuous) does not occur at the point where it originates. However, the onset of a shock wave inside the local supersonic zone contradicts the theorem mentioned above [1]. It is clear from physical considerations that the shock wave considered arises at the intersection of compression waves, which are incident on the closing part of the boundary of the local supersonic zone. Only one solution is known [9], which describes the intersection of compression waves in the formation of a shock wave inside a supersonic flow and which, as is also suggested below, possesses an infinite derivative of the intensity of the shock wave at the point where it originates. In these solutions it is not possible to satisfy all the above-mentioned conditions on the shock wave simultaneously, including (2.10). Only in the solutions in [6, 10] are these conditions completely satisfied, since the shock wave, to a first approximation, degenerated into a characteristic, i.e., it had a derivative of the intensity equal to zero. The gas-dynamic parameters on such a shock wave are continuous to a first approximation.

The above analysis shows that condition (2.10) can only be obtained by including terms $\sim y^{4m}$, the exact balance of which is not ensured in the self-similar approximation (2.7). Hence, the exact satisfaction of (2.10) on a shock wave of variable intensity which does not

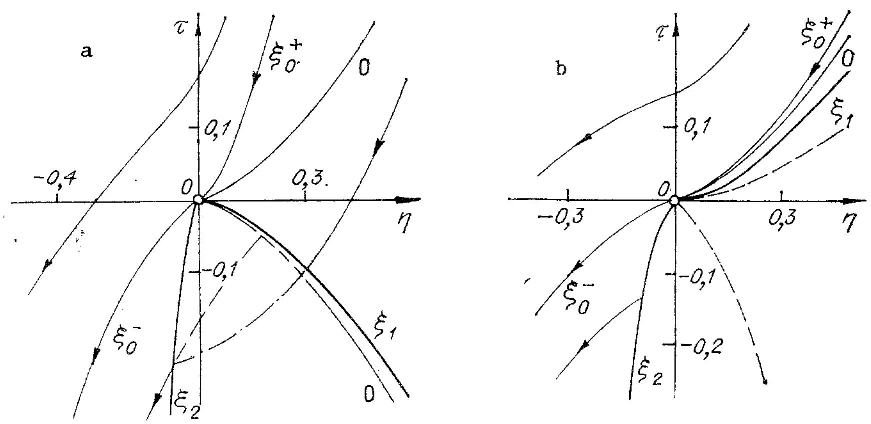

Fig. 2

degenerate into a characteristic has not been rigorously proved in the vicinity of the point where it is formed. Here we must estimate the solutions constructed for the least error, which is introduced by taking into account terms $\sim y^{4m}$. This estimate was made in [8], but with an inadmissible rejection of the condition [x] = 0 in favor of (2.10).

3. The proposed structure of the solution in the physical plane in the neighborhood of the point f (the origin of coordinates) is shown in Fig. 1. Figure 2 corresponds to the hodograph plane. Here the lines emerging from the origin of coordinates represent ξ = const (constant is indicated next to the line). Along the positive η axis the variable $\xi = -\infty$, along the negative axis $\xi = +\infty$, and along the τ axis, with which the sonic line coincides, $\xi = 1$. The equation of the characteristic $\xi = 0$. In the zone $-1 \leq \xi \leq 1$, $\tau \geq 0$ the hypergeometric function-solution is represented in the form of the sum of two linearly independent partial solutions of Eq. (2.1)

$$F(\xi) = c_1 F_1(\xi) + c_2 \xi^{2k+1/6} F_2(\xi),$$
$$F_1 = F(-k,\, 1/2 - k;\, 5/6 - 2k;\, \xi),\ F_2 = F(k + 1/6,\, k + 2/3;\, 2k + 7/6;\, \xi).$$

The functions F_1 and F_2 occurring in this expression only exist when $|\xi| \leq 1$, and provided that they are not equal to a negative integer of the expression $5/6 - 2k$ or $7/6 + 2k$. The factor in front of F_2 introduces a singularity onto the $\xi = 0$ characteristic if $2k + 1/6$ is not equal to a positive integer. However, F_1 does not exist for such k and, if $c_1 = 0$, we cannot have $\psi(0, \tau) = 0$. The use instead of F_1 in this case of another particular solution with a logarithmic singularity also does not satisfy the above condition of continuity of the derivatives of ψ on the C_+-characteristic (Fig. 1). Hence, by equating $c_2 = 0$ and $c_1 = -1$ we have a solution in which the self-similarity index k is not as yet determined:

$$\psi = -\tau^{2k} F_1(\xi),\ -1 \leqslant \xi \leqslant 1,\ \tau \geqslant 0,$$
$$\psi_\tau = -2\left[kF_1 + (1 - \xi) F_1'\right]\tau^{2k-1},\quad \psi_\eta = 3\left(\frac{2}{3}\right)^{2/3}(1 - \xi)^{2/3} F_1' \tau^{2k-2/3}. \tag{3.1}$$

The equation of the "zeroth" stream line, passing through the origin of coordinates, $\psi(\tau, \xi_0) = 0$ or $F_1(\xi_0) = 0$. Since $F_1(0) = 1$, in order that this line should lie between the sonic line and the characteristic when $\tau > 0$, as can be seen by comparing Figs. 1 and 2, it is necessary to satisfy the inequality (Γ is the gamma function)

$$F_1(1) = \frac{\Gamma(5/6 - 2k)\,\Gamma(1/3)}{\Gamma(5/6 - k)\,\Gamma(1/3 - k)} < 0.$$

Hence it follows that the region of permissible values for the self-similarity index is $5/12 > k - i/2 > 1/3$ ($i = 0, 1, \ldots$). The lower limit in this inequality is obtained from the condition for ψ_η to be finite as $\tau \to 0$, since otherwise the solution (3.1) would have a nonphysical singularity at the point f.

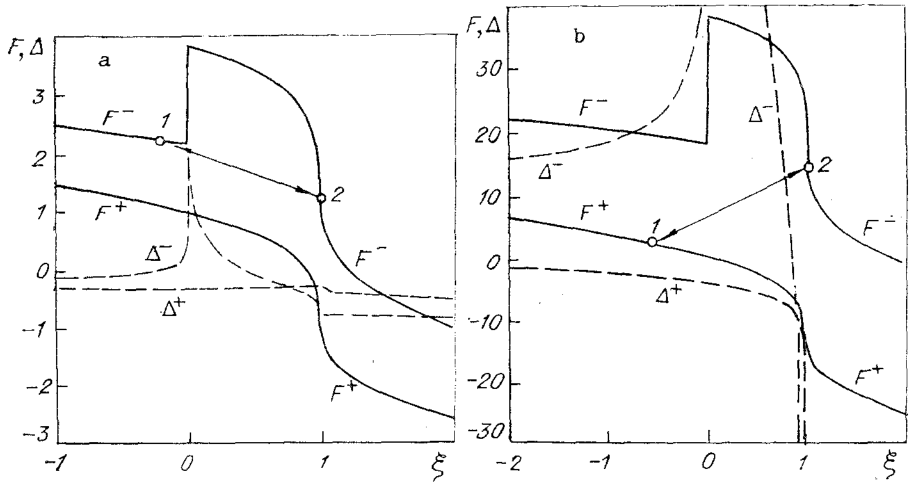

Fig. 3

Using the formulas for analytic continuation of the hypergeometric function [6], we obtain in the supersonic region of flow when $-\infty \leq \xi \leq -1$ and $-1 \leq \xi \leq 0$ for $\tau \leq 0$, respectively

$$\psi = -|\tau|^{2k}|\xi|^{k}[A_1F_3(\xi^{-1}) + B_1 \operatorname{sign}(\tau)|\xi|^{-1/2}F_4(\xi^{-1})], \tag{3.2}$$

$$F_3 = F(-k,\ 1/6 + k;\ 1/2;\ \xi^{-1}),\quad F_4 = F(1/2 - k,\ 2/3 + k;\ 3/2;\ \xi^{-1}),$$

$$A_1 = \frac{\pi^{1/2}\Gamma(5/6 - 2k)}{\Gamma(1/2 - k)\,\Gamma(5/6 - k)},\quad B_1 = -\frac{2\pi^{1/2}\Gamma(5/6 - 2k)}{\Gamma(-k)\,\Gamma(1/3 - k)};$$

$$\psi = |\tau|^{2k}[A_2F_1(\xi) + B_2|\xi|^{2k+1/6}F_2(\xi)], \tag{3.3}$$

$$A_2 = \frac{-1/2}{\sin[\pi(2k + 1/6)]},\quad B_2 = -\frac{\Gamma(5/6 - 2k)\,\Gamma(-2k - 1/6)}{\Gamma(-2k)\,\Gamma(2/3 - 2k)}\,2^{-4k-1/3}.$$

The hypergeometric function on the sonic line has a singularity for analytic continuation into the subsonic region $F'(\xi) \sim (1 - \xi)^{-2/3}$. Hence, by isolating this singularity, for $\xi \geq 1$ we obtain

$$\psi = |\tau|^{2k}\xi^{k}[a_1F_3(\xi^{-1}) - b_1 \operatorname{sign}(\tau)F_4(\xi^{-1})\xi^{-1/2}],$$
$$a_1 = 2A_1\sin[\pi(k - 1/6)],\ b_1 = 2B_1\cos[\pi(k - 1/6)]. \tag{3.4}$$

Finally, continuing this solution in the same way into the supersonic zone when $0 \leq \xi \leq 1$ and $\tau \leq 0$, we obtain

$$\psi = |\tau|^{2k}[a_2F_1(\xi) + b_2\xi^{2k+1/6}F_2(\xi)],$$
$$a_2 = -2A_2\sin[\pi(4k - 1/6)],\ b_2 = 2B_2\cos[\pi(2k + 1/6)]. \tag{3.5}$$

The two-valued nature of the solution with k from intervals with $i \geq 1$ can be shown by analyzing (3.2). In the supersonic region of the flow in this case the stream lines turn into the opposite direction. A solution with such an irregularity is clearly erroneous, as in [2], and hence we will further consider only the first interval for the index k, i.e., we will assume $i = 0$.

4. The hypergeometric function-solution calculated from (3.1)-(3.5) for $k = 0.3792$, is shown by the continuous lines in Fig. 3a for $-1 \leq \xi \leq 2$ (outside the figure these lines merge as $|\xi| \to \infty$), the curve F^+ corresponds in Fig. 2a to the region with $\tau > 0$, while the curve F^- is the solution in the lower half-plane. On the semicubic parabola $\xi = 0$ this function has a discontinuity

$$[F^-(0)] = \{1/2 + \sin[\pi(4k - 1/6)]\}/\sin[\pi(2k + 1/6)].$$

This discontinuity leads to a sudden jump in the stream function, which is shown in Fig. 2a by the dashed line. The Jacobian of the transformation Δ was calculated from (2.3) for fixed τ and is shown in Fig. 3 by the dashed line with the same indices as F. It can be

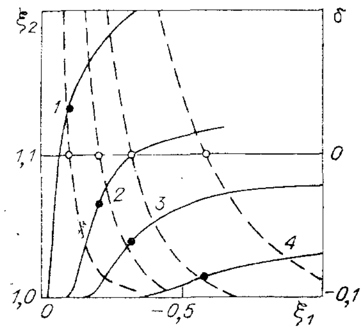

Fig. 4

seen that in the neighborhood of the characteristic with the discontinuity [F] the Jacobian Δ becomes positive, changing sign to the right and to the left of $\xi = 0$. In accordance with the theorem of the transformation, this indicates [6] a sudden change in the consideration, which is reflected into the region covering the zone with $\Delta \geq 0$.

The hypergeometric function shown in Fig. 3a also does not change qualitatively for other values of k from the first interval, but the dimensions of the zone with positive Jacobian turns out to depend considerably on k. Thus, whereas for k = 0.3792 this zone is situated at $-0.205 \leq \xi \leq 0.407$, as k increases the dimensions of the zone increase, and for k = 0.4075 its left limit $\xi = -\infty$. The right-hand boundary does not exceed $\xi = 1$. When k is increased further from the line $\Delta = 0$ in Fig. 3 Δ^+ and not Δ^- is intersected when $\xi < 0$, and the abscissa at the point of intersection increases. Figure 3b corresponds to this case. Points 1 and 2 of curve F^- in Fig. 3a are connected by means of the shock polar (the dash-dot line in Fig. 2a), but in this case the equation [x] = 0 is not satisfied, i.e., the thickness of the shock wave is not zero. If in this equation we eliminate $|\tau_1/\tau_2|$ using the relation $[\psi] = 0$, we obtain the equation $X(\xi_1, \xi_2) = 0$. It turns out that the points $\xi_1 < 0$ and $\xi_2 > 1$ do not exist on the curve F^-, for which the equation X = 0 would be satisfied. Hence, relation (2.8) is also not satisfied.

The above-mentioned behavior of the Jacobian (2.3) as $k \to 5/12$ indicates the need to search for a solution in which the point 1 belongs not to the curve F^-, as is shown in Fig. 3a, but to the left branch of the line F^+ in Fig. 3b. This indicates that on passing through the shock wave the angle of deviation of the flow changes sign from positive to negative. In fact, it can be seen from the calculations that when $0.3975 < k < 5/12$ in the plane of the variables ξ_1 and ξ_2 for each k there is a line $X(\xi_1, \xi_2) = 0$, on which, together with $[\psi] = 0$, the relation [x] = 0 is satisfied. In this case (2.10) is not satisfied exactly, but condition (2.8) is satisfied on each curve X = 0 when $\xi_2 \approx 1$.

In Fig. 4 the continuous lines 1-4 (X = 0) are drawn for k = 0.4158, 0.415, 0.4142, and 0.4125. For ξ_1 and ξ_2 belonging to the curve X = 0, we calculated the left-hand side of Eq. (2.8), which we denote by δ. The functions $\delta(\xi_1)$ are shown in Fig. 4 by the dashed curves. It can be seen that for each of the indicators considered, the dashed curve intersects the $\delta = 0$ axis. The points of intersection gives the required solution with a shockwave, on which, in the self-similar approximation considered, all the boundary conditions are satisfied (the mass, momentum, and energy flux equations). The Mach number behind the shock wave $M_2 < 1$ for $k > 0.4075$ and $M_2 > 1$ for $k < 0.4075$.

The hypergeometric function-solution is shown in Fig. 3b for k = 0.4125 with the same notation of the lines as in Fig. 3a. The flow hodograph in the vicinity of the point where the shock wave formed is shown in Fig. 2b. Here the zeroth stream lines correspond to the values $\xi_0{}^+ = 0.216$ and $\xi_0{}^- = 2.016$, while the boundaries of the sudden change $\xi_1 = -0.6010$ and $\xi_2 = 1.0138$. The Jacobian of the transformation is positive between the lines $\xi^+ = -3.464$ and $\xi^- = 0.864$ (the dashed curves in Fig. 2b). In the solution obtained the ratio of the angles of deviation of the flux on the shock wave is constant and equal to $\tau_1/\tau_2 = -6.5272$.

5. As already pointed out, the sudden change in compression, which bounds the local supersonic zone, is formed due to the action of the compression waves, which arrive from the sonic line on the closing parts of the zone boundary. In fact, in the solution obtained, as $k \to 5/12$, the zeroth stream line tends to a semicubic parabola $\xi_0 = 0$ in the upper half-

plane in Fig. 2b. Hence, in the region of the point f the supersonic flow approaches the flow in a simple compression wave, and in this case the formation of the shock wave precedes the formation of the beam of characteristics. The structure of the flow is similar to that obtained in the problem of supersonic flow around the concave surface of a wall [9].

Note that, as follows from Fig. 2b, the angle of deviation of the stream line at the point f (the origin of coordinates) has a minimum value compared with τ on the sonic line and on the "supersonic boundary" of the shock wave ξ_1. This behavior of the angle of inclination of the velocity vector agrees with the approximate solution [11], where it is stated that the occurrence of an irregularity of the waves on the closing part of the local supersonic zone indicates twisting of the supersonic flow, when the limit value of τ reached on the sonic line is higher than at the point where it intersects the wall of the body around which flow occurs.

In the above range the self-similarity index varies continuously, and hence the solution of the local problem of the formation of a shock wave is not unique. This is obviously confirmed physically by the fact that the intensity of the shock wave on the boundary of the local supersonic zone varies depending on the flow conditions, and changes solely in the scaling factor are insufficient – the exponent in Eqs. (2.7) also varies over a narrow range.

While remaining within the framework of the local problem we will determine the unique value k = 0.4017 from the minimum value of the error which is introduced by taking quadratic terms into account in (2.10). In this case on the shock wave $\xi_1 = -4.0922$, $\xi_2 = 0.9855$ and $\tau_1/\tau_2 = -1.8824$, and the zeroth stream lines correspond to $\xi_0^+ = 0.59$ and $\xi_0^- = 1.64$, $\Delta > 0$ when $-7.4007 \le \xi \le 0.77$.

Hence, in the solution constructed on the shock wave

$$\psi \sim [\eta]^{3k} \sim [p]^{3k},\ 3k \approx 6/5; \tag{5.1}$$

$$\omega = \omega_0 \psi^{\alpha},\ \alpha = (1-k)/k \approx 3/2. \tag{5.2}$$

Equation (5.2) for the vorticity ω was obtained from the relation for the increment in the entropy $[S] \sim [p]^3$, proved in [6]. The coefficient of proportionality ω_0 is found from the expansion of the Hugoniot adiabatic in series in powers of [S] and [p]. It follows from (5.1) that the intensity of the shock wave at the point where it originates has a derivative that is infinite along the length of the generatrix. Note that the intensity of the shock wave $[p] \sim \psi^{1/2}$ in the solution in [9].

After transforming the lines shown in Fig. 2b into the physical plane using (2.2) we obtain for the characteristic lines the equation $x = c_x|y|^n$, n = 1.404 with $c_x = -0.2593$ on the sonic line, $c_x = 0.3172$ on the shock wave, and $c_x = -0.8153$ on the characteristic arriving at the point f. These lines (the dashed, continuous, and dash-dot lines, respectively) are drawn in Fig. 1a. It can be seen that the sonic line and the characteristic c_+ are concave, while the shock wave is convex in the direction towards the incoming flow. At the point f the lines are tangent to one another and are perpendicular to the zeroth stream line (with the arrow).

The most probable solution is the one indicated above with minimum error corresponding to n = 1.4149, $c_x = 0.2958$ on the shock wave, $c_x = -0.8024$ on the C_+-characteristic, and $c_x = -0.7454$ and 0.3312 on the sonic line. These lines are drawn in Fig. 1b. It can be seen that the main difference from the solution shown in Fig. 1a is in the sonic line, which has a point of inflection f and limits the region of flow behind the shock wave. This shock wave relates to a weak family, since the Mach number behind it is greater than unity.

The solution obtained agrees with the theorems in [1] and explains, in the transonic approximation, the occurrence of a shock wave closing the local supersonic zone when a characteristic without a singularity arrives at the point of formation. This solution is an alternative to the conclusions in [4, 7, 8], but there is no basic for asserting that it is nonphysical, since the requirement that (2.9) or (2.10) should be exactly satisfied denotes a balance of the quadratic terms, which are clearly neglected in the class of self-similar transonic solutions. Hence, the determination of x' from (2.9) to obtain (2.10) is ill-posed, i.e., a small error in calculating $[\eta^2]$ leads to a considerable error in x' and, consequently, it is not possible to satisfy (2.10) exactly.

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THEORETICAL MODELS OF DETONATION OF A FLAT LAYER OF CONDENSED EXPLOSIVE WITH DIMINISHING DENSITY

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We have obtained in [1] the numerical nonself-similar solution of the problem of denotation wave (DW) propagation in a flat layer of a condensed explosive (EX) whose density ρ_0 diminishes according to a power law:

$$\rho_0 = \rho_{00}(1 - x/L_0)^\delta, \ \delta > 0. \tag{1}$$

Here, x is the present coordinate, ρ_{00} is the initial EX density at the x = 0 section, adjacent to an absolutely rigid wall, L_0 is the relative length over which ρ_0 formally vanishes, and δ is the exponent, which varies over the 0...2 range. The distribution of caloricity, i.e., of the specific energy release Q_0 per unit mass in the direction of thickness of the EX layer, was used in two limiting forms [2-4]:

$$Q_0 = Q_{00}(\rho_0/\rho_{00})^2; \tag{2}$$

$$Q_0 \equiv Q_{00} = \text{const}, \tag{3}$$

corresponding to either the purely elastic or the purely thermal character of the intrinsic energy of detonation products (DP) for a polytropic equation of state with the polytropic exponent k = 3 (Q_{00} is the caloricity corresponding to the density ρ_{00}). The DW behavior

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